

Weak magnetism in chiral quark models

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Abstract. We discuss symmetry breaking in the weak magnetism form factors for the semileptonic octet baryon decays. In the chiral quark model, the symmetry breaking can be accounted for in the masses and the quark spin polarizations can take on more general values due to Goldstone boson depolarization. Here we clarify some features of the chiral quark model prediction for the weak magnetism and compare to the corresponding result of the chiral quark soliton model.

1 Introduction

The weak interaction of the baryon octet has been a source of much information concerning the dynamics of the octet baryons. Especially the weak interactions can probe details of the hadronic structure at low energies.

Already early, the conserved vector current (CVC) hypothesis [1] extended to flavor SU(3) symmetry made predictions for the so called weak magnetism form factors.

In the quark model (QM), these predictions are easily rederived in the same limit. Introducing explicit SU(3) symmetry breaking through the masses of baryons and quarks, the QM will be able to model part of the symmetry breaking of the weak magnetism form factors [2, 3].

In the chiral quark model (χ QM) [4–6], it is also possible to include modifications of the spin polarizations of the quarks in these form factors.

In this note we discuss these form factors and compare to the same form factors derived in the so called chiral quark soliton model (χ QSM) [7]. The result of the latter model seems to differ in some respects with the result of the χ QM [8]. We here show that when linear terms in the symmetry breaking are eliminated the two models give the same predictions.

2 The weak form factors

The transition matrix element $\mathcal{M}_{B \rightarrow B' l^- \bar{\nu}_l}$ for the decay $B \rightarrow B' + l^- + \bar{\nu}_l$ ($q \rightarrow q' + l^- + \bar{\nu}_l$), is given by

$$\mathcal{M}_{B \rightarrow B' l^- \bar{\nu}_l} = \frac{G}{\sqrt{2}} V_{qq'} \langle B'(p') | J_{\text{weak}}^\mu | B(p) \rangle L_\mu, \quad (1)$$

where G is the Fermi coupling constant, $V_{qq'}$ is the qq' -element of the Cabibbo–Kobayashi–Maskawa mixing matrix, and L_μ is the leptonic current.

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The hadronic weak current is

$$J_{\text{weak}}^\mu = J_V^\mu - J_A^\mu, \quad (2)$$

where J_V^μ is the vector current and J_A^μ is the axial-vector current. The matrix element of the vector current in momentum space of the transition $B \rightarrow B' + l^- + \bar{\nu}_l$ is given by

$$\begin{aligned} \langle B'(p') | J_V^\mu | B(p) \rangle = & \bar{u}'(p') \left(f_1(q^2) \gamma^\mu \right. \\ & \left. - i \frac{f_2(q^2)}{M_B + M_{B'}} \sigma^{\mu\nu} q_\nu + \frac{f_3(q^2)}{M_B + M_{B'}} q^\mu \right) u(p), \end{aligned} \quad (3)$$

where M_B ($M_{B'}$), p (p'), $u(p)$ ($u'(p')$), and $|B(p)\rangle$ ($|B'(p')\rangle$) are the mass, momentum, Dirac spinor, and external baryon state of the initial (final) baryon B (B'), respectively, and $q = p - p'$ is the momentum transfer [9]. The functions $f_i(q^2)$, where $i = 1, 2, 3$, are the vector current form factors. These form factors are Lorentz scalars and contain information about the hadron dynamics. f_1 is the *vector* form factor, f_2 is the *induced tensor* form factor (or *weak magnetism* form factor or *anomalous magnetic moment* form factor), and f_3 is the *induced scalar* form factor.

In a previous paper [8], we have derived the form factors $f_i \equiv f_i(0)$, where $i = 1, 2, 3$, in the χ QM. Up to terms linear in the parameters $E \equiv \Delta/\Sigma$ and $\epsilon \equiv \delta/\sigma$, where $\Sigma = M_B + M_{B'}$, $\Delta = M_B - M_{B'}$, $\sigma = m_q + m_{q'}$, and $\delta = m_q - m_{q'}$, these form factors are

$$f_1 = f_1^{\text{QM}}, \quad (4)$$

$$f_2 = \left(\frac{\Sigma}{\sigma} G_A - 1 \right) f_1^{\text{QM}}, \quad (5)$$

$$f_3 = \frac{\Sigma}{\sigma} (E G_A - \epsilon) f_1^{\text{QM}}. \quad (6)$$

Here $G_A \equiv g_1^{\text{QM}}/f_1^{\text{QM}}$, with $f_1^{\text{QM}} \equiv \langle B' | \lambda_{qq'} \otimes 1 | B \rangle$ and $g_1^{\text{QM}} \equiv \langle B' | \lambda_{qq'} \otimes \sigma^z | B \rangle$. The $\lambda_{qq'}$ is the SU(3) matrix that

effectuates the flavor transition and the σ^z operator measures the spin polarizations of the quarks in the baryons.

2.1 The weak axial-vector form factors

The weak axial-vector form factors $G_A = g_1^{\text{QM}}/f_1^{\text{QM}}$ can be obtained from the SU(6) QM expressed in terms of the parameters F and D [10]. In the χ QM, the G_A 's are expressed in the quark spin polarizations of the proton, *i.e.* Δu , Δd , and Δs . These spin polarizations differ considerably from the ones in the SU(6) QM due to the depolarization of the quark spins by the Goldstone bosons (GBs). The spin polarizations in the χ QM are calculated with one GB emission. They are [6]

$$\Delta u = \frac{4}{3} - \frac{37}{9}a, \quad (7)$$

$$\Delta d = -\frac{1}{3} - \frac{2}{9}a, \quad (8)$$

$$\Delta s = -a, \quad (9)$$

where a is the parameter which measures the probability of emission of a GB from a quark. Using the relations $F = \frac{1}{2}(\Delta u - \Delta s)$ and $D = \frac{1}{2}(\Delta u - 2\Delta d + \Delta s)$ [11], we have

$$G_A^{np} = \Delta u - \Delta d, \quad (10)$$

$$G_A^{\Sigma^- \Sigma^0} = \frac{1}{2}(\Delta u - \Delta s), \quad (11)$$

$$g_1^{\text{QM}\Sigma^\pm \Lambda} = \frac{1}{\sqrt{6}}(\Delta u - 2\Delta d + \Delta s), \quad (12)$$

$$G_A^{\Xi^- \Xi^0} = \Delta d - \Delta s \quad (13)$$

for the $\Delta S = 0$ decays and

$$G_A^{\Sigma^- n} = \Delta d - \Delta s, \quad (14)$$

$$G_A^{\Xi^- \Sigma^0} = \Delta u - \Delta d, \quad (15)$$

$$G_A^{\Xi^- \Lambda} = \frac{1}{3}(\Delta u + \Delta d - 2\Delta s), \quad (16)$$

$$G_A^{\Lambda p} = \frac{1}{3}(2\Delta u - \Delta d - \Delta s), \quad (17)$$

$$G_A^{\Xi^0 \Sigma^+} = \Delta u - \Delta d \quad (18)$$

for the $\Delta S = 1$ decays.

The magnetic moments of the octet baryons and the weak axial-vector form factor G_A^{np} can be used to fit the parameter a and the quark magnetic moment μ_d . Using $\mu_u = -2\mu_d$ and $\mu_s = 2\mu_d/3$ [12], we then obtain $a \simeq 0.104$ and $\mu_d \simeq -1.196 \mu_N$. This gives $\Delta u \simeq 0.90$, $\Delta d \simeq -0.36$, and $\Delta s \simeq -0.10$.

In the χ QM, the effective quark masses can be determined from the fitted value of μ_d . The quark masses are then $m_u^{\text{eff}} = m_d^{\text{eff}} = m^{\text{eff}} \approx 260 \text{ MeV}$ and $m_s^{\text{eff}} = 3m^{\text{eff}}/2 \approx 390 \text{ MeV}$. In the following, we will use $m_q \equiv m_q^{\text{eff}}$, where $q = u, d, s$.

The values of the $G_A^{BB'}$'s for the χ QM are listed in Table 1, where for reference also the axial-vector form factors of the naive QM (NQM) are displayed.

Table 1. Weak axial-vector form factors, $G_A^{BB'}$. The values in the NQM column are the SU(6) values for the weak axial-vector form factors and the values in the χ QM column are obtained from the quark spin polarizations. $g_1^{\text{QM}\Sigma^\pm \Lambda}$ are given instead of $G_A^{\Sigma^\pm \Lambda}$, since $f_1^{\text{QM}\Sigma^\pm \Lambda} = 0$. The experimental values for $G_A^{BB'}$ have been obtained from [17], except for the $g_1^{\text{QM}\Sigma^- \Lambda}$ and $G_A^{\Xi^- \Sigma^0}$ values, which are CERN WA2 [18, 19] results from branching ratio measurements

Quantity	Experimental value	NQM	χ QM
G_A^{np}	1.2670 ± 0.0035	$\frac{5}{3}$	1.26
$G_A^{\Sigma^- \Sigma^0}$	-	$\frac{2}{3}$	0.50
$g_1^{\text{QM}\Sigma^- \Lambda}$	0.589 ± 0.016	$\sqrt{\frac{2}{3}}$	0.62
$g_1^{\text{QM}\Sigma^+ \Lambda}$	-	$\sqrt{\frac{2}{3}}$	0.62
$G_A^{\Xi^- \Xi^0}$	-	$-\frac{1}{3}$	-0.25
$G_A^{\Sigma^- n}$	-0.340 ± 0.017	$-\frac{1}{3}$	-0.25
$G_A^{\Xi^- \Sigma^0}$	1.25 ± 0.15	$\frac{5}{3}$	1.26
$G_A^{\Xi^- \Lambda}$	0.25 ± 0.05	$\frac{1}{3}$	0.25
$G_A^{\Lambda p}$	0.718 ± 0.015	1	0.76
$G_A^{\Xi^0 \Sigma^+}$	-	$\frac{5}{3}$	1.26

3 The ratio ρ_f and the “weak magnetism”

We will now concentrate on the “weak magnetism” form factor ρ_f , which is defined as

$$\rho_f \equiv \frac{f_2}{f_1}. \quad (19)$$

Inserting (4) and (5) in (19), we obtain

$$\rho_f = \frac{\Sigma}{\sigma} G_A - 1. \quad (20)$$

Since there are no linear terms in E and ϵ in either f_1 or f_2 , the formula for ρ_f is valid up to terms of second order in E and ϵ . The quark masses in this and related formulas appear as effective masses, and the parametric dependence of the quark spin polarization Δq , where $q = u, d, s$, on the emission probability a of GBs incorporates effects of relativistic corrections and other possible dynamical effects on both the magnetic moments [13] and the ρ_f 's. When these effects are taken into account directly, in terms of a changed structure of the currents, the fits become worse [14].

The expression (20) for ρ_f above is closely related to the corresponding formula for the magnetic moments μ_B of the octet baryons used in earlier studies. In the same approximation as here, we have

$$f_1 = Q_B, \quad (21)$$

$$f_2 = \Sigma \mu_B - Q_B = \Sigma \sum_{q=u,d,s} \frac{e_q}{2m_q} \Delta q - Q_B$$

$$\equiv \frac{M_B}{M_N} \kappa_B, \quad (22)$$

where e_q is the quark charge, $Q_B = 0, \pm 1$ is the charge of the baryon, and κ_B is the anomalous magnetic moment of the baryon in nuclear magnetons. It is therefore in principle possible to convert expression (20) above to an expression in terms of the magnetic moments. This will eliminate the parametric model dependence. In the following, we will discuss how this can be done in a way that preserves the absence of terms linear in E and ϵ in (20).

3.1 The weak magnetism and CVC

From SU(3) flavor symmetry the weak magnetism form factors can be related to the magnetic moments of the nucleons. The result is [10]

$$\rho_f^{np} = (\mu(p) - \mu(n))/\mu_N - 1, \quad (23)$$

$$\rho_f^{\Sigma^- \Sigma^0} = (\mu(p) + \frac{1}{2}\mu(n))/\mu_N - 1, \quad (24)$$

$$f_2^{\Sigma^\pm \Lambda} = -\sqrt{\frac{3}{2}}\mu(n)/\mu_N, \quad (25)$$

$$\rho_f^{\Xi^- \Xi^0} = (\mu(p) + 2\mu(n))/\mu_N - 1, \quad (26)$$

$$\rho_f^{\Sigma^- n} = (\mu(p) + 2\mu(n))/\mu_N - 1, \quad (27)$$

$$\rho_f^{\Xi^- \Sigma^0} = (\mu(p) - \mu(n))/\mu_N - 1, \quad (28)$$

$$\rho_f^{\Xi^- \Lambda} = (\mu(p) + \mu(n))/\mu_N - 1, \quad (29)$$

$$\rho_f^{\Lambda p} = \mu(p)/\mu_N - 1, \quad (30)$$

$$\rho_f^{\Xi^0 \Sigma^+} = (\mu(p) - \mu(n))/\mu_N - 1. \quad (31)$$

This is called the (extended) CVC hypothesis. In the NQM, using the formula (20) above, all these relations emerge by putting $\Sigma/\sigma = M_N/m$ and using $\Delta u = 4/3$, $\Delta d = -1/3$, and $\Delta s = 0$. Then all ρ_f 's can be expressed in terms of the quark magnetic moment $\mu_d = -\mu_u/2 = -1/(6m)$, which can be related to the proton and neutron magnetic moments.

It is, however, obvious that the symmetry breaking in the masses, neither of the quarks nor of the baryons are then accounted for. In particular, $\mu_s = \mu_d$ in this approximation.

On the next level of refinement, one could therefore try to use in (20) instead the real baryon masses together with $m_s/m = 3/2$, along with the SU(6) QM values for the spin polarizations G_A . Since the magnetic moments are fairly well accounted for in the NQM, this is probably a rather good improvement. In the χ QM, we also allow the spin polarizations to deviate from their SU(6) values, increasing the improvement still somewhat. The results are given in Table 2.

3.2 The weak magnetism in the chiral quark model.

The $\Delta S = 0$ cases

The formula (20) above is transformed into an expression in terms of the magnetic moments of the baryons, when G_A/σ is expressed in the magnetic moments through $\Delta q/(2m_q)$.

Table 2. The ratios $\rho_f^{BB'} \equiv \frac{f_2^{BB'}}{f_1^{BB'}}$. The experimental values have been obtained from [18] (see also [19]). $f_2^{\Sigma^\pm \Lambda}$ are given instead of $\rho_f^{\Sigma^\pm \Lambda}$, since $f_1^{\Sigma^\pm \Lambda} = 0$

Quantity	Experimental value	CVC	χ QM	χ QSM [7]
ρ_f^{np}	3.71 ± 0.00 (input)	3.71	3.53	3.71
$\rho_f^{\Sigma^- \Sigma^0}$	-	0.84	1.31	1.30
$f_2^{\Sigma^- \Lambda}$	3.52 ± 3.52	2.34	2.73	2.80
$f_2^{\Sigma^+ \Lambda}$	-	2.34	2.72	2.80
$\rho_f^{\Xi^- \Xi^0}$	-	-2.03	-2.27	-
$\rho_f^{\Sigma^- n}$	-1.78 ± 0.61	-2.03	-1.82	-1.67
$\rho_f^{\Xi^- \Sigma^0}$	-	3.71	3.85	3.61
$\rho_f^{\Xi^- \Lambda}$	-0.44 ± 0.46	-0.12	-0.06	0.10
$\rho_f^{\Lambda p}$	2.43 ± 1.49	1.79	1.38	1.52
$\rho_f^{\Xi^0 \Sigma^+}$	-	3.71	3.83	-

Consider for example the $n \rightarrow p + l^- + \bar{\nu}_l$ decay. We can then show, using $\mu(p) = \Delta u \mu_u + \Delta d \mu_d + \Delta s \mu_s$ and the corresponding formula for $\mu(n)$, that

$$\begin{aligned} \rho_f^{np} &= \frac{1}{2} \left(1 + \frac{M_n}{M_p} \right) (\mu(p) - \mu(n)) \frac{1}{\mu_N} - 1 \\ &\simeq (\mu(p) - \mu(n)) \frac{1}{\mu_N} - 1 = \kappa_p - \kappa_n. \end{aligned} \quad (32)$$

Here we have used the expression $G_A^{np} = \Delta u - \Delta d$ from Subsection 2.1 above and $\mu_u = -2\mu_d$. Equation (32) is exactly the CVC formula for the $n \rightarrow p + l^- + \bar{\nu}_l$ decay.

For the other transitions among the octet baryons, the χ QM predicts the symmetry breaking in these weak magnetic moments due to the symmetry breaking in the masses both of quarks and baryons. In the following, we study the symmetry breaking using isospin symmetry and begin with the $\Delta S = 0$ transitions.

For the $\Sigma^- \rightarrow \Sigma^0$ transition, we have

$$\rho_f^{\Sigma^- \Sigma^0} = \frac{2M_\Sigma}{2m} \frac{1}{2} (\Delta u - \Delta s) - 1. \quad (33)$$

Using the expressions for $\mu(\Sigma^-)$ and $\mu(\Sigma^+)$, we find in this case

$$\Delta u - \Delta s = \frac{1}{3\mu_d} (\mu(\Sigma^-) - \mu(\Sigma^+)), \quad (34)$$

which leads to

$$\begin{aligned} \rho_f^{\Sigma^- \Sigma^0} &= M_\Sigma (\mu(\Sigma^+) - \mu(\Sigma^-)) - 1 \\ &= \frac{M_\Sigma}{2M_N} (\kappa_{\Sigma^+} - \kappa_{\Sigma^-}). \end{aligned} \quad (35)$$

In a similar way, we obtain for the $\Xi^- \rightarrow \Xi^0$ transition

$$\begin{aligned} \rho_f^{\Xi^- \Xi^0} &= 2M_\Xi (\mu(\Xi^0) - \mu(\Xi^-)) - 1 \\ &= \frac{M_\Xi}{M_N} (\kappa_{\Xi^0} - \kappa_{\Xi^-}). \end{aligned} \quad (36)$$

Direct computation in the χ QM using the formula (20) with the parameters Δq , where $q = u, d, s$, gives $\rho_f^{\Xi^- \Xi^0} \approx -2.27$, whereas the formula (36) above gives $\rho_f^{\Xi^- \Xi^0} \approx -1.84$ when the experimental magnetic moments are inserted. The discrepancy is due to the relatively poor agreement between the χ QM prediction of the magnetic moments of the Ξ^0 and Ξ^- and their experimental values.

For $f_1 = 0$, we calculate instead $f_2 = \Sigma g_1^{QM} / \sigma$. This replaces ρ_f for $\Sigma^\pm \rightarrow \Lambda$. The result can be expressed in terms of the $\Sigma\Lambda$ magnetic moment transition matrix element:

$$\mu(\Sigma\Lambda) = -\frac{1}{2\sqrt{3}}(\Delta u - 2\Delta d + \Delta s)(\mu_u - \mu_d). \quad (37)$$

We then obtain

$$f_2^{\Sigma^+\Lambda} = -\sqrt{2}(M_\Lambda + M_\Sigma)\mu(\Sigma\Lambda). \quad (38)$$

In all the cases above, there is an inherent ambiguity in the choice of magnetic moments, since in the SU(6) QM the form factors G_A can be expressed in only two polarization differences, say $\Delta u - \Delta d$ and $\Delta u - \Delta s$. In our approximation, the different choices are related by the sum-rule

$$\mu(p) - \mu(n) + \mu(\Sigma^-) - \mu(\Sigma^+) + \mu(\Xi^0) - \mu(\Xi^-) = 0, \quad (39)$$

which follows under quite general assumptions on the spin polarizations and the magnetic moments of the quarks, and in particular from the SU(6) QM. This sum-rule is valid to within about $0.5 \mu_N$ on the left hand side.

In these cases, $\sigma = 2m$ and the quark magnetic moments $\mu_u = -2\mu_d = 1/(3m)$ are related to $1/\sigma$ without any symmetry breaking. When we pass to anomalous magnetic moments in (32), (35), and (36) it is no longer possible to use (39), since extra linear terms in E will then appear.

3.3 The $\Delta S = 1$ cases

The cases with $\Delta S = 1$ are less straightforward, and there is no ‘‘natural’’ way to express the spin polarizations in terms of the magnetic moments, since many different possibilities give the same formal result. To begin with, there is a complication that $\sigma = m + m_s$ in these cases. The crucial factor in the transformation of (20) into an expression in terms of magnetic moments is, up to normalization, given by an expression of the form

$$A \simeq \frac{1}{\sigma(\mu_d + x\mu_s)}, \quad (40)$$

where x is a real parameter. This expression must not contain terms linear in the small quantity $\epsilon = \delta/\sigma$. It is easy to see that the condition for this is given by $x = 1$.

Let us study this for the case of the $\Sigma^- \rightarrow n$ transition. We have

$$\rho_f^{\Sigma^- n} = \frac{M_\Sigma + M_N}{m + m_s} \frac{\mu(\Sigma^-) - \mu(n)}{\mu_s + 2\mu_d} - 1. \quad (41)$$

This can be rewritten as

$$\rho_f^{\Sigma^- n} = (M_\Sigma + M_N)A_1(\mu(n) - \mu(\Sigma^-)) - 1, \quad (42)$$

where $A_1 = -1/((m + m_s)(\mu_s + 2\mu_d))$. It is easy to check that this expression is linear in ϵ , since $x = 1/2$. In fact, using $m_s = 3m/2$, we get $A_1 = 9/10$, so the deviation from 1 is 10%.

An alternative way of obtaining the spin polarization for the $\Sigma^- \rightarrow n$ transition is to use

$$\begin{aligned} \mu_{\Sigma n} &\equiv \mu(n) + \frac{1}{2}\mu(p) - \mu(\Sigma^-) - \frac{1}{2}\mu(\Sigma^+) \\ &= -\frac{3}{2}(\Delta d - \Delta s)(\mu_d + \mu_s). \end{aligned} \quad (43)$$

This gives

$$\rho_f^{\Sigma^- n} = (M_\Sigma + M_N)A_2\mu_{\Sigma n} - 1, \quad (44)$$

where

$$A_2 = -\frac{2}{3(m + m_s)(\mu_s + \mu_d)} \simeq 1 + \mathcal{O}(\epsilon^2).$$

In fact, for $m_s = 3m/2$, we obtain $A_2 = 24/25$, which is only 4% from 1. In the following, this term will therefore be put equal to 1. We can then write

$$\rho_f^{\Sigma^- n} = \frac{M_\Sigma + M_N}{2M_N}\kappa_{\Sigma n}, \quad (45)$$

where $\kappa_{\Sigma n} \equiv \kappa_n + \frac{1}{2}\kappa_p - \kappa_{\Sigma^-} - \frac{1}{2}\kappa_{\Sigma^+}$. Next consider

$$\begin{aligned} \rho_f^{\Xi^- \Sigma^0} &= \frac{M_\Xi + M_\Sigma}{m + m_s}(\Delta u - \Delta d) - 1 \\ &= (\mu(p) - \mu(n))\frac{1}{\mu_N} - 1 = \kappa_p - \kappa_n, \end{aligned} \quad (46)$$

where we have used $M_\Xi + M_\Sigma \simeq 3(m + m_s)$. This expression happens to coincide exactly with CVC (see (28)).

However, to neglect the hyperfine interaction in the mass formulas for the baryons means to discard terms linear in the hyperfine interaction constant, which is generally of the order 50 MeV. This is almost of the same order as the symmetry breaking mass difference δ between the quark masses. We should therefore not be satisfied with this approximation.

Again, it is possible to use another combination to express the axial-vector coupling constant. This is given by

$$\begin{aligned} \mu_{\Xi\Sigma} &\equiv \mu(\Sigma^+) + \frac{1}{2}\mu(\Sigma^-) - \mu(\Xi^0) - \frac{1}{2}\mu(\Xi^-) \\ &= -\frac{3}{2}(\Delta u - \Delta d)(\mu_d + \mu_s). \end{aligned} \quad (47)$$

Using this gives

$$\rho_f^{\Xi^- \Sigma^0} = (M_\Xi + M_\Sigma)\mu_{\Xi\Sigma} - 1. \quad (48)$$

For later use this can be rewritten as

$$\rho_f^{\Xi^- \Sigma^0} = \frac{M_\Xi + M_\Sigma}{2M_N}(\kappa_{\Sigma^+} + \frac{1}{2}\kappa_{\Sigma^-} - \kappa_{\Xi^0} - \frac{1}{2}\kappa_{\Xi^-}). \quad (49)$$

Since all G_A 's can be expressed in terms of the spin polarization differences $\Delta d - \Delta s$ and $\Delta u - \Delta d$ it is possible to express all other $\Delta S = 1$ transitions in terms of $\mu_{\Sigma n}$ and $\mu_{\Xi\Sigma}$. However, if we want to convert the result from magnetic moments to anomalous magnetic moments, we must also avoid terms that are linear in the mass ratios $E = \Delta/\Sigma$.

Consider therefore next

$$\rho_f^{\Xi^- \Lambda} = \frac{1}{3} \frac{M_\Xi + M_\Lambda}{m + m_s} (\Delta u + \Delta d - 2\Delta s) - 1. \quad (50)$$

To avoid terms that are linear in E , we must in this case use

$$\begin{aligned} \mu_{\Xi\Lambda} &\equiv \mu(p) + 3\mu(\Lambda) - \mu(\Xi^0) - \frac{1}{2}\mu(\Sigma^+) - \frac{5}{2}\mu(\Xi^-) \\ &= -\frac{3}{2}(\Delta u + \Delta d - 2\Delta s)(\mu_d + \mu_s). \end{aligned} \quad (51)$$

We then obtain, in the same approximation,

$$\rho_f^{\Xi^- \Lambda} = (M_\Xi + M_\Sigma) \frac{1}{3} \mu_{\Xi\Lambda} - 1 = \frac{M_\Xi + M_\Sigma}{2M_N} \frac{1}{3} \kappa_{\Xi\Lambda} + A_3, \quad (52)$$

where $\kappa_{\Xi\Lambda} \equiv \kappa_p + 3\kappa_\Lambda - \kappa_{\Xi^0} - \frac{1}{2}\kappa_{\Sigma^+} - \frac{5}{2}\kappa_{\Xi^-}$ and $A_3 = \frac{1}{3}(M_\Sigma + M_\Xi)(1/(2M_N) - 1/(4M_\Sigma) + 5/(4M_\Xi)) - 1$. It is easy to verify that $A_3 = \mathcal{O}(E^2)$ and may therefore be neglected.

Finally, we can express ρ_f^{Ap} in terms of magnetic moments in the same way starting from

$$\rho_f^{Ap} = \frac{1}{3} \frac{M_\Lambda + M_N}{m + m_s} (2\Delta u - \Delta d - \Delta s) - 1. \quad (53)$$

The best way of expressing the form factor uses

$$\begin{aligned} \mu_{\Lambda p} &\equiv \mu(n) + \frac{5}{2}\mu(p) + \frac{1}{2}\mu(\Sigma^-) - 3\mu(\Lambda) - \mu(\Xi^-) \\ &= -\frac{3}{2}(2\Delta u - \Delta d - \Delta s)(\mu_d + \mu_s). \end{aligned} \quad (54)$$

Omitting second order mass differences, this gives

$$\rho_f^{Ap} = (M_\Lambda + M_N) \frac{1}{3} \mu_{\Lambda p} - 1 = \frac{M_\Lambda + M_N}{2M_N} \frac{1}{3} \kappa_{\Lambda p}, \quad (55)$$

where $\kappa_{\Lambda p} \equiv \kappa_n + \frac{5}{2}\kappa_p + \frac{1}{2}\kappa_{\Sigma^-} - 3\kappa_\Lambda - \kappa_{\Xi^-}$.

3.4 The weak magnetism in the chiral quark soliton model

The weak magnetic form factors have also been calculated in the χ QSM. The result, after normalization in our con-

vention, is given in [7] as

$$\rho_f^{np} = \kappa_p - \kappa_n, \quad (56)$$

$$\rho_f^{\Sigma^- \Sigma^0} = \frac{M_\Sigma}{2M_N} (\kappa_{\Sigma^+} - \kappa_{\Sigma^-}), \quad (57)$$

$$f_2^{\Sigma^\pm \Lambda} = -\sqrt{2} \frac{M_\Sigma + M_\Lambda}{2M_N} \frac{\mu_{\Sigma\Lambda}}{\mu_N}, \quad (58)$$

$$\begin{aligned} \rho_f^{\Sigma^- n} &= \frac{M_\Sigma + M_N}{2M_N} \\ &\times (\kappa_n + \frac{1}{2}\kappa_p - \kappa_{\Sigma^-} - \frac{1}{2}\kappa_{\Sigma^+}), \end{aligned} \quad (59)$$

$$\begin{aligned} \rho_f^{\Xi^- \Sigma^0} &= \frac{M_\Xi + M_\Sigma}{2M_N} \\ &\times (\kappa_{\Sigma^+} + \frac{1}{2}\kappa_{\Sigma^-} - \kappa_{\Xi^0} - \frac{1}{2}\kappa_{\Xi^-}), \end{aligned} \quad (60)$$

$$\begin{aligned} \rho_f^{\Xi^- \Lambda} &= \frac{M_\Xi + M_\Lambda}{2M_N} \frac{1}{3} \\ &\times (\kappa_p + 3\kappa_\Lambda - \kappa_{\Xi^0} - \frac{1}{2}\kappa_{\Sigma^+} - \frac{5}{2}\kappa_{\Xi^-}), \end{aligned} \quad (61)$$

$$\begin{aligned} \rho_f^{Ap} &= \frac{M_\Lambda + M_N}{2M_N} \frac{1}{3} \\ &\times (\kappa_n + \frac{5}{2}\kappa_p + \frac{1}{2}\kappa_{\Sigma^-} - 3\kappa_\Lambda - \kappa_{\Xi^-}). \end{aligned} \quad (62)$$

We have here neglected the possible change in the transition from the anomalous magnetic moments to the full magnetic moments that might be related to the change in normalization. By this, we mean that in the above expressions, $\kappa_B = \mu_B/\mu_N - Q_B M_N/M_B$, and the normalization, of course, is of relevance when the symmetry is broken.

However, our understanding is that, apart from a factor of $M_B/(M_B + M_{B'})$, these differences in our normalization are of the order $\mathcal{O}(m_s^2)$ or $\mathcal{O}(m_s/N_c)$, *i.e.* in terms that are anyhow neglected in the above formulas [7], and the corresponding terms of second order or higher in the mass ratios that are neglected in our calculations.

From Sect. 3 it should be clear then, that the method of expressing ρ_f , that avoids introducing linear terms in the symmetry breaking masses, in general produces expressions that coincide with those above. This shows that the χ QM and the χ QSM give the same results when linear terms in the symmetry breaking are eliminated.

4 Discussion

The particular choice of combinations of magnetic moments, that enables one to express the ρ_f 's in term of anomalous magnetic moments, are enforced from the cancellation of linear terms in both the quark and baryonic mass differences. The analysis presented here shows that when this is done the χ QM and the χ QSM give the same results. The earlier noticed numerical differences are related to the difficulty to reproduce the octet baryon magnetic moments in the χ QM without symmetry breaking in the spin polarizations. This can be seen *e.g.* in the case of Ξ^0 and Ξ^- .

It is of course possible to stop and be satisfied at the level where the ρ_f 's are expressed in terms of magnetic moments. Then, since all G_A 's can be expressed in terms

of only two spin polarization differences, there are several equivalent relations for the ρ_f 's related to sum rules for the G_A 's. On top of that, there is in this case also the possibility to use the sum-rules for the magnetic moments to find alternative ways to express the ρ_f 's.

For the ρ_f 's the existing experimental data is given in Table 2.

Let us consider this table.

The CVC values listed are in a way half experimental results, since they use the measured values of the anomalous magnetic moments for the nucleons as input data to calculate these values.

Since the magnetic moments of the quarks are fitted to the magnetic moments of the proton and neutron, the SU(6) QM results should coincide with CVC in the absence of symmetry breaking and are not listed.

All values obtained for the ρ_f 's in the χ QM lie within the experimental errors, where experimental data exist. (The experimental results have large errors, though.)

In one case, that of neutron decay, we can see that $\rho_f(\chi\text{QM}) \approx \rho_f(\text{CVC})$. For the other decays, the ρ_f 's of the χ QM incorporate effects of vector current non-conservation due to the mass differences between the isomultiplets as well as depolarization of the spin due to GB emission.

All calculated values for the χ QM have the same sign as the CVC values and they are also close in magnitude. The numerical results cannot be expected to be much better than within 10%. Already isospin is violated to a few percent.

For comparison, we have in two cases calculated the ρ_f 's obtained by neglecting the hyperfine interaction, since it is rather small. The results are

$$\rho_f^{\Xi^- \Sigma^0} = (\mu(p) - \mu(n))/\mu_N - 1 \simeq 3.71$$

and

$$\begin{aligned} \rho_f^{\Xi^- \Lambda} &= (\mu(\Sigma^+) + \mu(\Xi^0) - \mu(\Xi^-) - \mu(\Sigma^-))/(3\mu_N) - 1 \\ &= 0.01 \pm 0.02, \end{aligned}$$

which both are very close to the values using (49) and (52), respectively.

5 Summary and conclusions

We have studied the baryonic weak magnetism form factors in detail in the spirit of the χ QM and compared the results with the χ QSM. The comparison shows that the results are in good agreement, and that the differences are of the order of reliability of the results in all cases. This might indicate that the main part of the symmetry breaking is accounted for in these formulas. The numerical results are presented in Tables 1 and 2.

The present investigation has used the SU(3) symmetric coupling in the χ QM and the static approximation for the quarks. A natural improvement would be to incorporate lowest order non-static effects and further SU(3) symmetry breaking effects [15,16], to obtain better agreement with experimental data. In particular, we expect that this would lead to a closer agreement with the ρ_f ratios obtained from direct application of (20), since symmetry breaking can better account for the octet baryon magnetic moments [12]. SU(3) symmetry breaking also leads to better agreement for g_A^{np} [12,15,16].

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